Physics for Game Programmers: Spatial Data Structures

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"I feel like a million tonight …
…But one at a time."

Mae West

Collision Queries

- Find pairs of objects that are colliding now, …
- … or will collide over the next frame if no counter action is taken.
- Compute data for response.

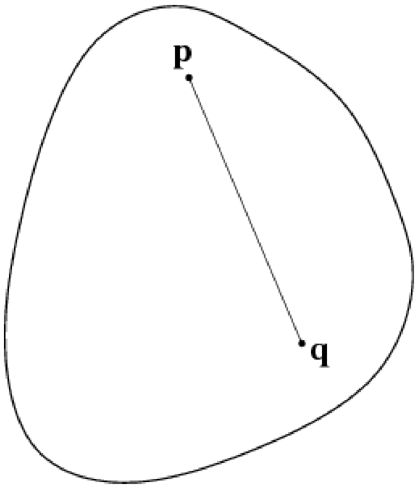
Continuous Collision Queries

- Fast moving objects, such as bullets and photons (for visibility queries), need to be swept in order to catch all potential hits.
- Querying a scene by casting rays (or shapes) is an important ability.

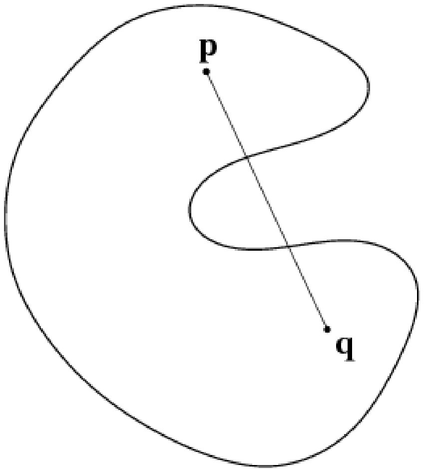
Collision Objects

- Static environment (buildings, terrain) is typically modeled using polygon meshes.
- Moving objects (player, NPCs, vehicles, projectiles) are typically convex shapes.
- We focus on convex-mesh collisions. Mesh-mesh is hardly ever necessary.

Convex Shapes



Convex



Concave

Three Phases

- **Broad phase**: Determine all pairs of independently moving objects that potentially collide.
- **Mid phase**: Determine potentially colliding primitives in complex objects.
- **Narrow phase**: Determine contact between convex primitives.

Mid Phase

- Complex objects such as triangle meshes may be composed of lots of primitives.
- Testing all primitives will take too long, especially if only a few may be colliding.
- How to speed things up? Or rather, achieve a more output-sensitive solution?

Spatial Coherence

- Expresses the degree in which a set of primitives can be ordered based on spatial location.
- Primitives occupy only a small portion of the total space.
- A location in space is associated with a limited number of primitives.

Which Is More Coherent?

Divide & Conquer

Capture spatial coherence by using some divide & conquer scheme:

- **Space Partitioning**: subdivide space into convex cells.
- **Model Partitioning**: subdivide a set of primitives into subsets and maintain a bounding volume per subset.

Uniform Grid

- Subdivide a volume into uniform rectangular cells (voxels).
- Cells need not keep coordinates of their position.
- Position (x, y, z) goes into cell

$$(i, j, k) = \left(\lfloor x / e_x \rfloor, \lfloor y / e_y \rfloor, \lfloor z / e_z \rfloor \right)$$

where e_x, e_y, e_z are the cell dimensions.

Uniform Grid (cont'd)

Two alternative strategies:

- Add a primitive to all cells that overlap the prim's bounding box. Overlapping boxes must occupy the same cell.
- Add a primitive to the cell that contains the center of its box. Neighboring cells need to be visited for overlapping boxes, but cells contain fewer objects.

Uniform Grid (cont'd)

- Grids work well for large number of primitives of roughly equal size and density (e.g. cloth, fluids).
- For these cases, grids have O(1) memory and query overhead.

Spatial Hashing

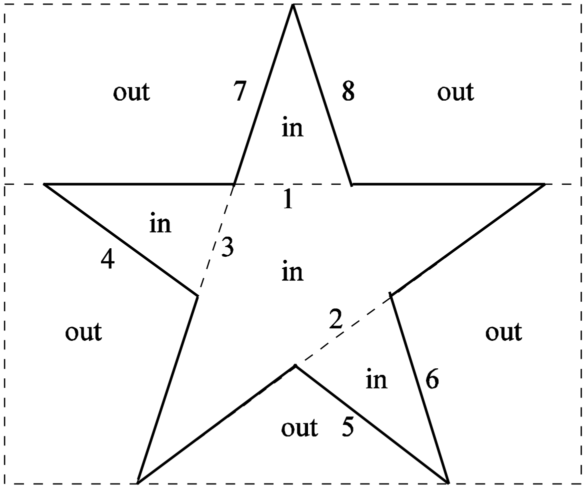
- Same as uniform grid except that the space is unbounded.
- Cell ID (i, j, k) is hashed to a bucket in a hash table.
- Neighboring cells can still be visited. Simply compute hashes for ($i\pm1$, $j\pm1$, $k\pm1$).

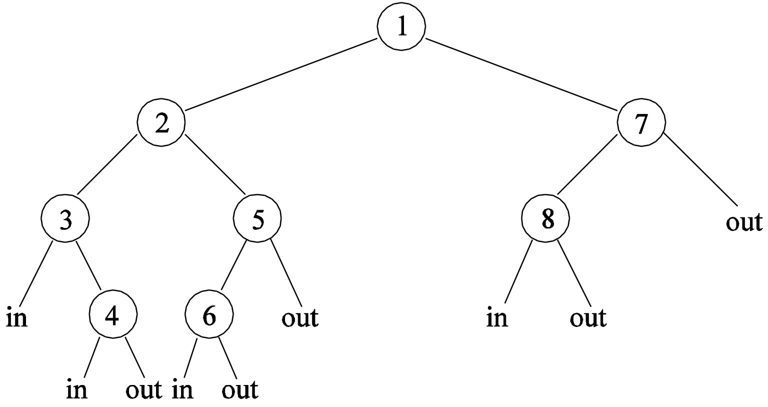
Spatial Hashing (cont'd)

- As for grids, spatial hashing only works well for primitives of roughly equal size and density.
- Remote cells are hashed to the same bucket, so exploitation of spatial coherence may not be that great.

Binary Space Partitioning

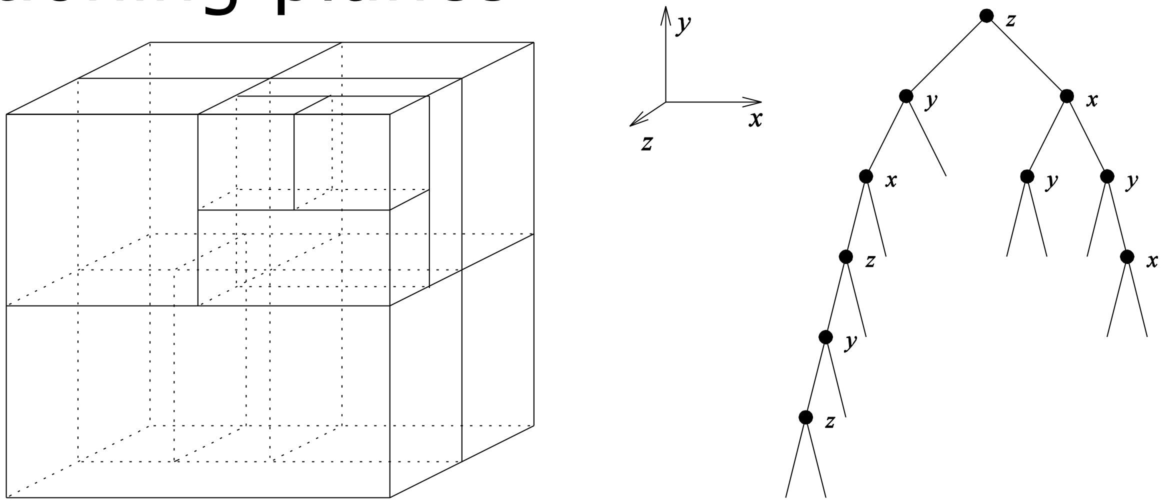
- A BSP is a hierarchical subdivision of space into convex cells by (hyper)planes.





k-D Trees

- A *k*-D trees is a BSP with axis-aligned partitioning planes.



BSP Trees versus *k*-D Trees

- *k*-D trees have smaller memory footprints and faster traversal times per node.
- BSP Trees need fewer nodes to “carve out” a proper volume representation.
- The volume enclosed by a polygon mesh can be accurately represented by a BSP.

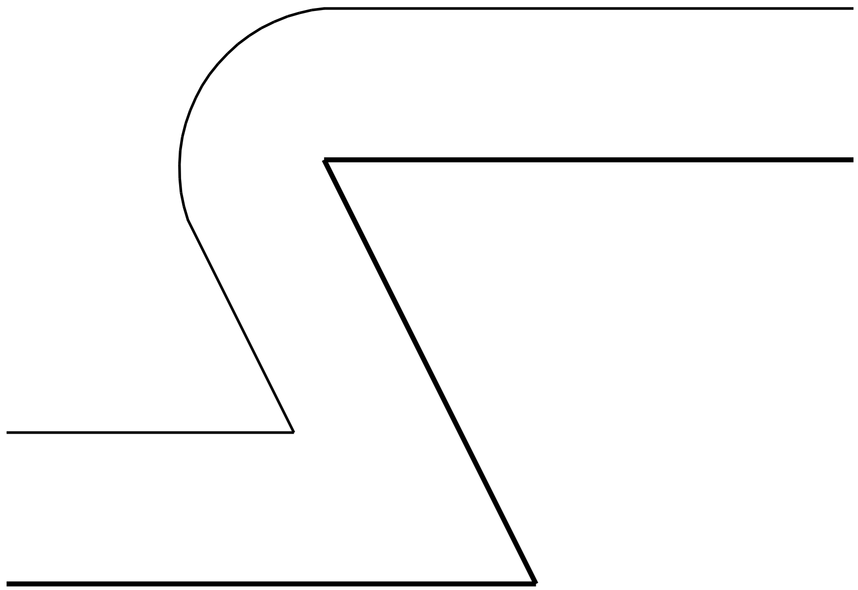
Point Queries on a BSP

```
while (!node->isLeaf) {
    node = node->plane.above(p) ?
           node->posChild :
           node->negChild;
}
return node->inside;
```

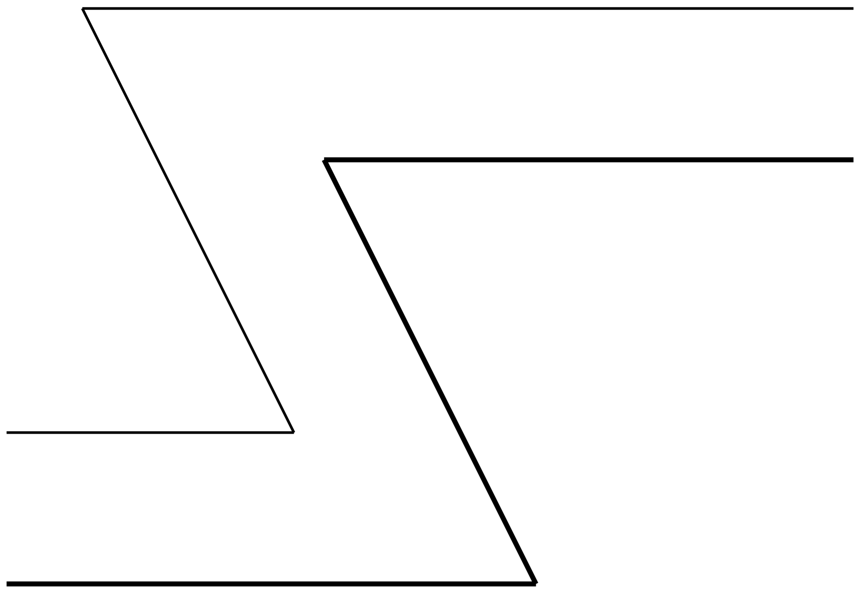
Volume Queries on a BSP

- Fat objects may overlap multiple cells, so multiple root paths need to be traversed.
- Shrink the query object to a point and dilate the environment by adding the negate of the query object [Quake2].
- The dilated environment is better known as *Configuration Space Obstacle* (CSO).

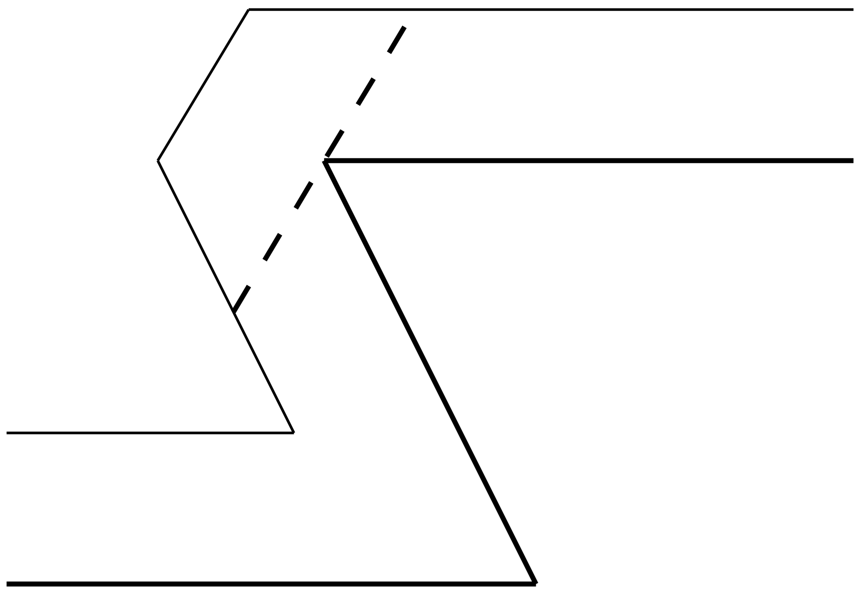
Exact CSO for a Disk



Approximate CSO for a Disk



Add Auxiliary Bevel Planes



Dynamic Plane-shift BSP Trees

- What if we have query objects with different sizes?
- Keep the original BSP and offset the planes while traversing [Melax, MDK2].
- Since obstacles occur both in the + and - half-space, we offset in both directions.

Bounding Volumes

- Should fit the model as tightly as possible.
- Overlap tests between volumes should be cheap.
- Should use a small amount of memory.
- Cost of computing the best-fit bounding volume should be low.

Bounding Volumes (cont'd)

Good bounding volume types are:

- Spheres
- Axis-aligned bounding boxes (AABBs)
- Discrete-orientation polytopes (k-DOPs)
- Oriented bounding boxes (OBBs)

Bounding Volume Types

	Fit	Test (ops)	Memory (scalars)	Best-fit Cost
Sphere	poor	11	4	high
AABB	fair	≤ 6	6	low
k-DOP	good	$\leq 2k$	$2k$	medium
OBB	excellent	≤ 200	15	high

Why AABBs?

- Offer a fair trade-off between storage space and fit.
- Overlap tests are fast.
- Allow for fast construction and update of bounding volumes.
- Storage requirements can be further reduced through compression.

Binary AABB Tree

- Each internal node has two children.
- Each leaf node contains one primitive.
- For N primitives we have N leaf nodes and N – 1 internal nodes (2N – 1 AABBs in total).
- Best trees need not be, and usually are not, height-balanced.

Volume Queries on an AABB Tree

- Compute the volume's AABB in the AABB tree's local coordinate system.
- Recursively visit all nodes whose AABBs overlap the volume's AABB.
- Test each visited leaf's primitive against the query volume.

Sequential Tree Traversal

```
int i = 0;
while (i != nodes.size()) {
    if (overlap(queryBox, nodes[i].aabb)) {
        if (nodes[i].isLeaf)
            primTest(nodes[i]);
        ++i;
    } else i += nodes[i].skip;
}
```

Memory Considerations

- Nodes are stored in a single array.
- Each left child is stored immediately to the right of its parent.
- The skip field stores the number nodes in the subtree (the number of nodes to skip if the overlap test is negative).

Ray Cast on an AABB Tree

- Returns the primitive that is stabbed by the ray earliest.
- Requires a line-segment-versus-box test.
- Each stabbed primitive shortens the ray.
- Traverse the AABB tree testing the AABB closest to the ray’s source first.

Line-segment vs. Box Test

- Use a SAT testing the box's three principal axes and the three cross products of principal axes and the line segment.
- If the line segment is almost parallel to an axis then the cross product is close to zero and the SAT may return false negatives!!!

Line-segment Box Test (cont'd)

- The cross product rejection test has the form

$$| \mathbf{r}_z * \mathbf{c}_y - \mathbf{r}_y * \mathbf{c}_z | > |\mathbf{r}_z| * \mathbf{e}_y + |\mathbf{r}_y| * \mathbf{e}_z,$$

where **r** is the ray direction, and **c** and **e** are resp. the center and extent of the AABB.

- The rounding noise in the lhs can get greater than the rhs, resulting in a false negative!!

Line-segment Box Test (cont'd)

- The lhs suffers from cancellation, if **c** is at some distance from the origin.
- The solution is to add an upper bound for the rounding noise to the rhs, which is

$$\max(|\mathbf{r}_z * \mathbf{c}_y|, |\mathbf{r}_y * \mathbf{c}_z|)\varepsilon,$$

Here, ε is the machine epsilon of the number type.

Shape Casting on an AABB Tree

- Similar to ray casting but now we need to find the primitive that is hit by a convex shape.
- Perform a ray cast on the Minkowski sum of the node's AABB and the shape's box:

$$[a, b] - [c, d] = [a - d, b - c]$$

AABB Tree Construction

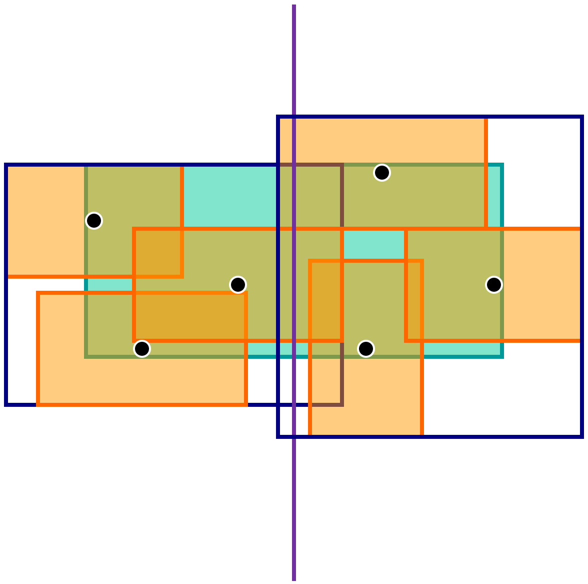
- AABB Trees are typically constructed top down.
- Bottom-up construction may yield good trees, but takes a lot of processing.
- Start with a set of AABBs of the primitives.

Top-down Construction

- Compute the AABB of the set of AABBs. This is the root node’s volume.
- Split the set using a plane. The plane is chosen according to some heuristic.
- AABBs that straddle the plane are added to the dominant side. (AABBs of the two sets may overlap.)
- Repeat until all sets contain one AABB.

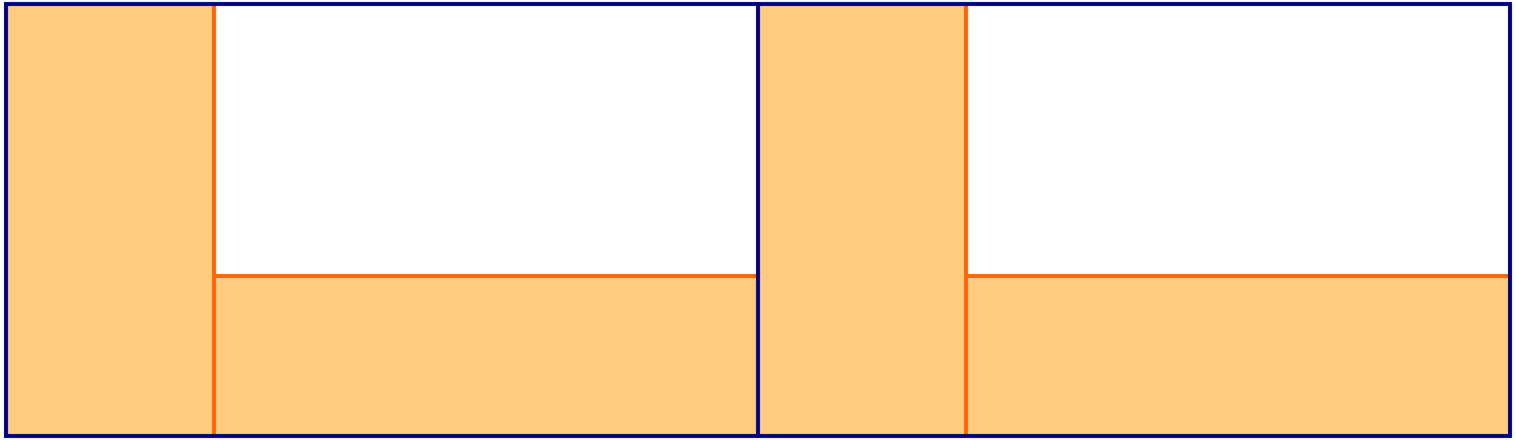
Median Split Heuristic

- Compute the bounding box of the set of AABB center points.
- Choose the plane that splits the box in half along the longest axis.



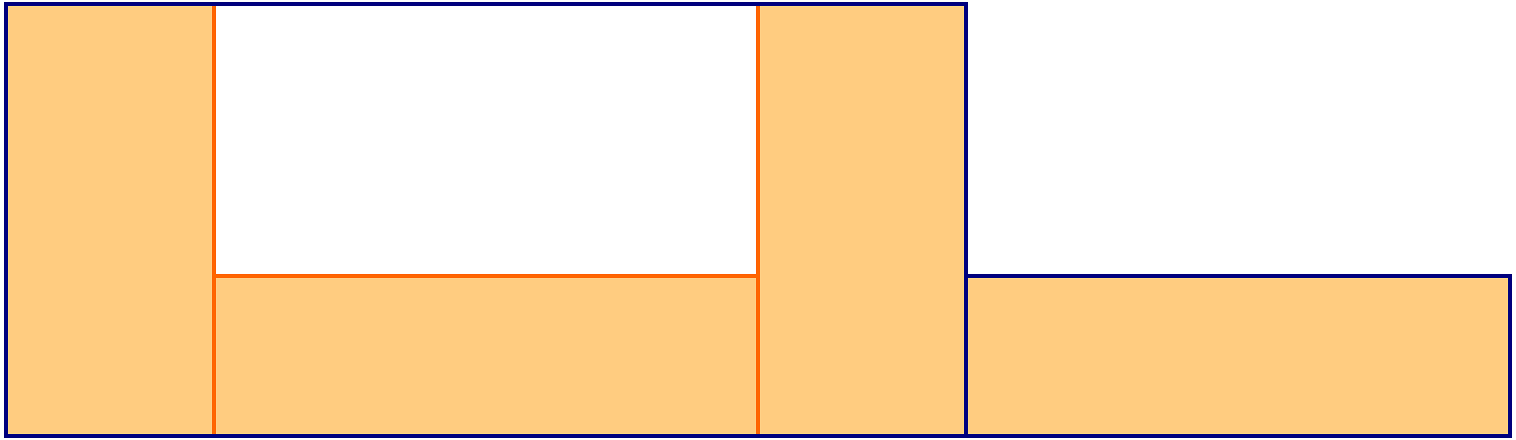
Median Split Heuristic No Good

- Median splits may not carve out empty space really well.



Better Splitting Heuristic

- Off-center splits may do better.



Surface Area Heuristic

- Find the splitting plane that minimizes

$$SA(AABB_{left}) * N_{left} + SA(AABB_{right}) * N_{right}$$

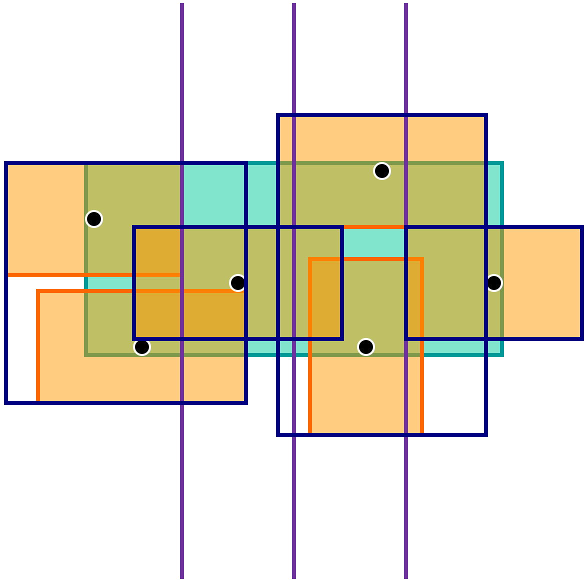
- Here, SA(AABB) is the surface area of the box, and N is the number of primitives.

Surface Area Heuristic (cont'd)

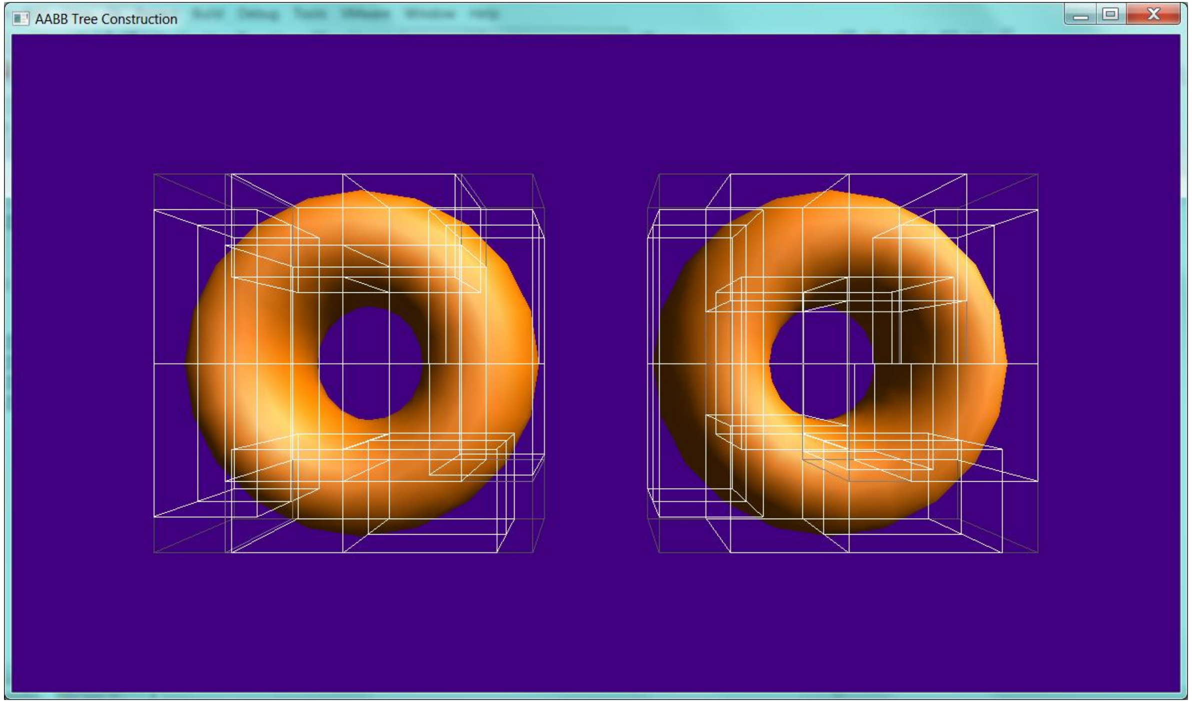
- Determining the best SAH splitting plane can be computationally expensive.
- Sufficiently-good splitting planes can be found quickly by using a binning technique:
- Evaluate a pre-defined set of splitting planes, and pick the best one.

Surface Area Heuristic (cont'd)

- Group primitives per cell and compute the AABB of the group.
- Compute $AABB_{left}$ and $AABB_{right}$ for each splitting plane from these cell AABBs, and compute their SA.



Top-down Construction Demo



Updating AABB Trees

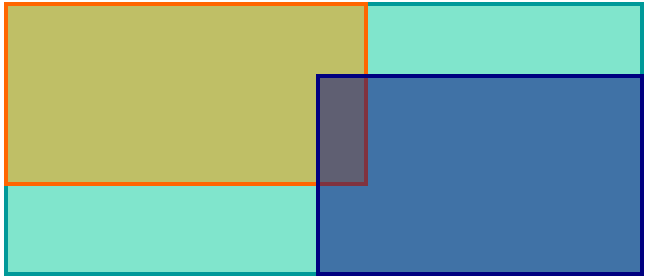
- AABB trees can be updated rather than reconstructed for deformable meshes.
- First recompute the AABBs of the leaves.
- Work your way back to the root:
A parent’s box is the AABB of the children’s boxes.
- For extreme deformations reconstruction is better and may not be that slow [Wald].

Compressed AABB Trees [Gomez]

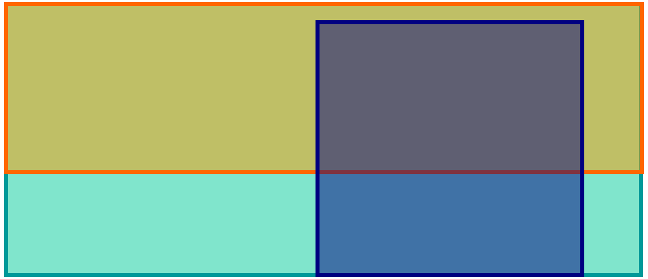
- Only 6 of the 12 faces of the child AABBs differ from the parent's faces.
- Pack the children paired, and only store the coordinates of these new inner faces.
- For each of the 6 inner faces store a bit to denote which child it belongs to.

Compressed AABB Trees [Gomez]

- Inner faces are shared among children.



- Sharing need not be even.

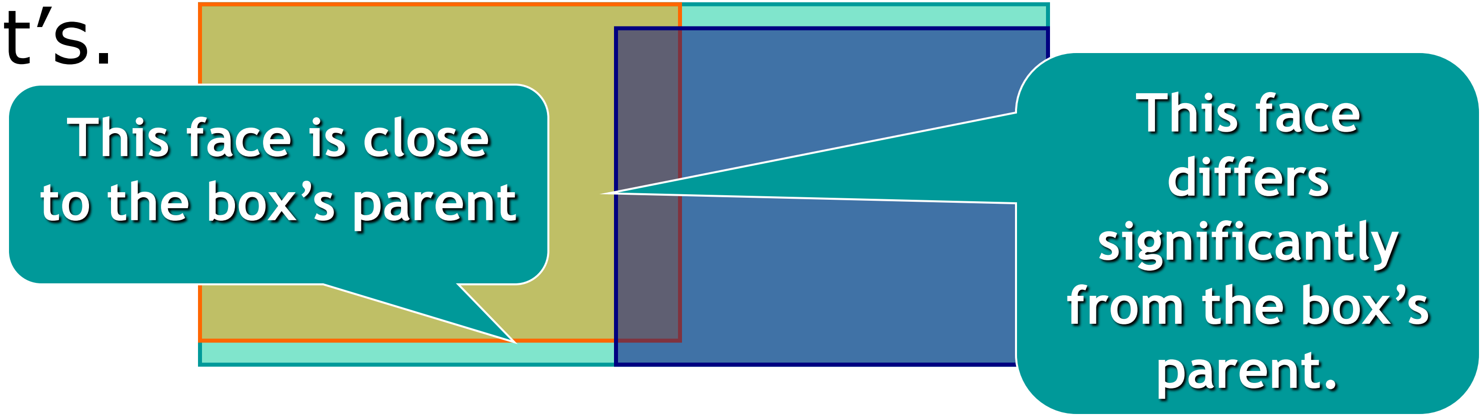


Compressed AABB Trees [Gomez]

- Encode each of the 6 inner face coordinates by a single byte.
- The byte represents the coordinate as a fraction of the parent’s AABB.
- Lower bounds are rounded down. Upper bounds are rounded up.
- This reduces the memory footprint from 56 to 16 bytes per primitive.

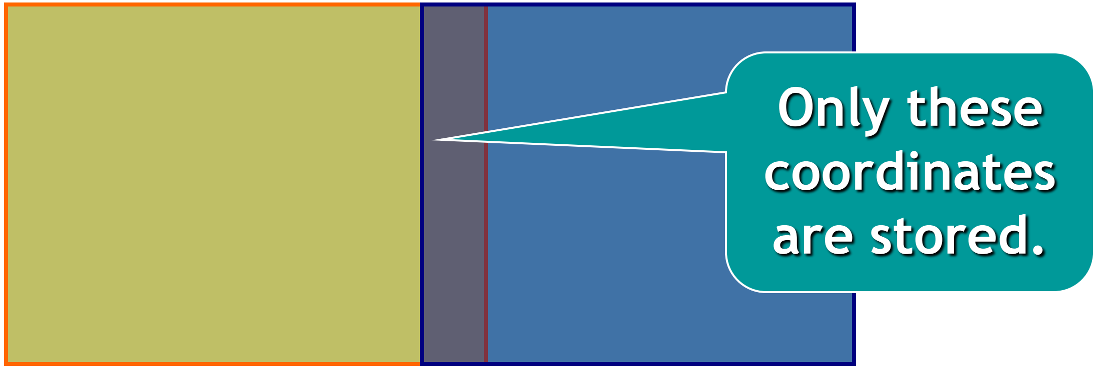
Boxtree [Zachmann]

- Since the set of primitives is split along the longest axis, only one of each child's faces will differ significantly from its parent's.



Boxtree [Zachmann]

- Store only the coordinate for the inner faces (similar to *k*-d tree.)
- The other coordinates are inherited from the parent box.



Boxtree [Zachmann]

The Boxtree (aka Bounding-Interval Hierarchy) has a few benefits over traditional AABB trees:

- Smaller memory footprint.
- Slightly faster build times.
- Faster traversal times due to the fact that the number of axes in the SAT can be further reduced.
- Empty space is captured less greedily, so YMMV!

References

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- Gino van den Bergen. Efficient Collision Detection of Complex Deformable Models using AABB Trees. *JGT, Vol. 2, No. 4,* 1997
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- Ingo Wald. On Fast Construction of SAH Based Bounding Volume Hierarchies. *Proc. Eurographics*, 2007
- Miguel Gomez. Compressed Axis-Aligned Bounding Box Trees. *Game Programming Gems 2*, 2001
- Gabriel Zachmann. Minimal Hierarchical Collision Detection. *Proc. VRST, 2002*.

Thank You!

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